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Study on two-stage investment decision-making for PPP projects based on binomial tree option algorithm

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Abstract. This paper applies the theories of real options and incomplete contract to discuss two-stage investment decision-making for PPP projects. Combined with binomial tree option algorithm, the decision models of private investors have been further discussed, considering external costs. And taking a sewage treatment project in central China as an example, an empirical analysis is made. The results indicate that: 1) the private investors tend to delay expansion of project without regard to external costs; 2) by introducing appropriate penalty coefficient or changing the ownership of the project, the action of private investors to harm the public interest can be suppressed, so as to achieve ecological compensation.

Key words. Binomial tree option, incomplete contract, two-stage, penalty coefficient.

1. Introduction

Public and private partnership (PPP) is a long-term partnership established in the field of infrastructure and public services. The essence is to provide public goods or services through the establishment of a new partnership contract relationship, under the rules of cooperation and operational mechanisms.PPP projects are usually large-scale investment, long construction period, and covering a wide range. And the researches of risk and risk allocation have always been the hot spot in the field [1-4].

There are two main approaches to the risk sharing and allocation of PPP project risk. One is to evaluate the risks and expected benefits for each stage of the project

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life cycle as much as possible, through a series of models, the risk of each stage of the project is evaluated as much as possible, and the expectations of each stage of the project life cycle are quantified in consideration of the corresponding risks, so that the participants can make optimal decision in advance. Real options [5-7], game theory and the combination of them have been recently hot topics in the research. In most studies, the life of project is divided into the planning period, the operation period and the transfer period, which is advantageous to calculate option value, but ignores the uncertainty of the period division. Another is to consider the risk sharing of PPP projects from the perspective of incomplete contracts [8-9], given that it is unrealistic to avoid all risks through complex prior contractual arrangements. However, in the incomplete contract theory, the calculation of the net income function of GHM model depends on the true value of the project (Grossman and Hart, 1986; Hart and Moore, 1990). But, most of the existing studies consider static value rather than the option value caused by the uncertainty of the project. Previous studies are mostly static values and less consideration of the option value brought by project uncertainty.

This paper focuses on two-stage investment decision-making for PPP projects, which has a clear division of the construction phase and the uncertainty depending on the actual changes of the project traffic in the second phase of the project construction period. The main jobs of this paper are embodied in the following: (1) When a full option is given, the private investors' decision-making behavior have been analyzed, on the basis of combining the real option theory and incomplete contract theory; (2) In the case of internalization of external costs, the changes in the decision-making behavior of private investors after the introduction of the penalty coefficient are studied.

2. Analysis of option characteristics and incomplete contract in PPP project

The option is a right, not an obligation. Compared to traditional static net present value (NPV), real options help decision makers decide when to invest. So this elastic value that carries risks is also more practical [10-12].PPP projects are often partially or completely irreversible, and there are complex and diverse risks in the implementation process, which provide a suitable platform for the dynamic decision-making mechanism of real options. Especially, for the projects with phased expansion and renovation, there is a clear expansion option decision-making situation. And the introduction of real options in the contract can reduce the distortion of value under NPV evaluation system and avoid the higher negotiating costs.

Owing to the existence of the foresee costs, contracting costs and confirm costs, the incomplete contract theory has become a regular research topic in the field of economics, and has been gradually introduced into the field of law, political philosophy and other fields. Because of long duration and subject diversification, the political, economic and social environment are complex and varied in the process, which have made it difficult for the participants to consider in advance and conclude a complete contract. Instead, the participants are more willing to renegotiate to solve any problems. The main topic of the incomplete contract is to discuss how the arrangements should be made to minimize loss of efficiency, if the incomplete contract results in an invalid investment. This paper has attempted to provide a contract support for the participants to provide the basis for the renegotiation of the second phase of the project, that is, the stipulation of the triggering mechanism of the relevant compensation clause have been arranged to prevent the loss of social benefits caused by the private investors in pursuit of the maximum unilateral interests.

In general, the value evaluation of project, based on the real options is more reflect the risk value during the operation period. And this method provides a reasonable basis for the participants to complete the contract or negotiate again, besides, it supports the realization of maximum entire benefit of the PPP projects.

3. Two-stage investment decision-making model of PPP projects

3.1. Assumptions

(1)Only the government and the social capital are participants in the PPP projects, and such as financing institutions, the public and other third parties are not included. Meanwhile, the public interest is represented by the government;

(2)The agreement of PPP projects is an incomplete contract; therefore, not all contingency factors can be expected.

(3)The project is divided into two phases for construction; wherein, the second phase of the project can be completed independently, or merged with the first phase construction;

(4)In the process of investment decision-making, learning costs, conversion costs and other factors are not considered.

3.2. Two - stage investment decision - making model that considers real options only

Common real option evaluation methods are binomial tree and B-S (Black-Scholes). The former is suitable for discrete time option pricing and the latter is for continuous option pricing. In the two-stage investment projects, there are two investment strategies for private investors, one is to carry out the first phase of construction immediately and wait for the opportunity to determine when to carry out the second phase, and another is to carry out two phases directly. Therefore, the main consideration in this model is the impact of delayed option.

(1)Binomial tree model

The binomial tree option pricing model is a classical numerical approach to option pricing proposed by Cox, Ross, and Rubinstein in 1979. The basic idea of the binomial tree model is to use discrete models to simulate the continuous movement of asset prices, and then use the mean and variance to determine the relevant parameters. Finally, the rational price of the current options is deduced backwards from the end of the binomial tree graph.

In general, the price of the option is calculated as:

$$f = e^{-r\bullet\Delta t}\left[p\bullet Q_t^u + (1-p)\bullet Q_t^d\right] \quad (1)$$

Where,r is discount rate, $p$is probability of growth and $1-p$is probability of falling, Q_t^u is actual traffic when it goes up,Q_t^d is actual traffic when it goes down.

(2)Expectation of traffic

The uncertainty is derived from the oscillations in the traffic that rise or fall, and the oscillation is determined by the volatility. Assume that the volatility of each stage is the square root of the interval length, which can be regarded as a discrete form of Geometric Brownian motion. There are:

$$u = (1+s)^t(1+\sigma\bullet\Delta t^{1/2}) \quad (2)$$

$$d = (1+s)^t(1-\sigma\bullet\Delta t^{1/2}) \quad (3)$$

Here, $\Delta$is the annual volatility,Δt is the unit interval,s is the expectation of traffic growth rate,u is the size of up-movement,d is the size of down-movement.

(3)Actual traffic

Set traffic cap is $Q_t^{\max}$ and traffic guarantee is Q_t^{gua} in period $t$then, the actual traffic isQ_t:

$$Q_t = \begin{cases} Min(Q_t^{\max}, Q_{t-\Delta t}\cdot u)t\in[0,T], if\ up-movement\ and\ Q_{t-\Delta t}\cdot u > Q_t^{gua} \\ Min(Q_t^{\max}, Q_{t-\Delta t}\cdot d), t\in[0,T], if\ down-movement\ and\ Q_{t-\Delta t}\cdot d > Q_t^{gua} \\ Q_t^{gua} t\in[0,T], Q_t \le Q_t^{gua} \end{cases} \quad (4)$$

Where, $T$is the concessionary period. When the first phase of construction is completed, there is$Q_t^{\max} = Q_t^{1st}$, and when the second phase of construction is completed, there is$Q_t^{\max} = Q_t^{2nd}$.

(4)Net income of project

The traffic can be either up when decision window comes to period $t+\Delta t$from period$t$.The revenue value CI_t is:

$$CI_t = \sum(Q_t^u\bullet p + Q_t^d\bullet(1-p))\bullet\lambda \quad (5)$$

Where,λ is revenue per traffic (The operating costs per traffic have been deducted). Then, the net income of project is:

$$RI = \sum_{t=0}^{T}(CI_t\cdot(1+r)^{-t}) \quad (6)$$

$$Con = \sum Con_t\cdot(1+r)^{-t} \quad (7)$$

$$NI = RI - Con \quad (8)$$

RI is total revenue and Con is total construction cost over the life cycle,Con_t is the cost of project in period t.Private investors would like to choose the best net income scenario from each program as the best.

3.3. Two-stage investment decision-making model that considers real options and incomplete contract

PPP was originally designed to solve the problem of poor government financing and inefficient public projects management. To achieve the Pareto optimal of the public goods market, the factors of production and products must have the value of "completeness". That is, the price or cost should consider its externalities. When the project capacity or processing capacity exceeds the upper limit, the private investors tend to delay the expansion of the project because of the absence of external costs.

Assume that the external cost is expressed by the penalty coefficient, set as$c$($c > 0$). When the actual traffic exceeds the upper limit of the processing capacity, the penalty$c$ per unit on the excess will be levied to compensate for the resulting external influences that if the private investor do not carry out the expansion, in the case of expansion potential. The total penalty isCO_t^{pen}:

$$CO_t^{pen} = \begin{cases} 0 & Q_t \le Q_t^{1st} \\ (Q_t - Q_t^{1st}) \cdot \lambda \cdot c & Q_t^{1st} < Q_t \le Q_t^{2nd} \\ (Q_t^{2nd} - Q_t^{1st}) \cdot \lambda \cdot c & Q_t > Q_t^{2nd} \end{cases} \tag{9}$$

In the case of a penalty coefficient, the net income of the project is:

$$RI' = \sum_{t=0}^{T} (CI_t - CO_t^{pen})(1+r)^{-t} \tag{10}$$

$$NI' = RI' - Con \tag{11}$$

4. Case study

4.1. Background and parameters

A certain airport has a sewage station, of which the daily scale of sewage treatment is 5,500 tons. In order to meet the need of long-term sewage treatment, the BOT model is planned for the expansion. The expansion project will be divided into two stages, and the daily sewage treatment capacity of each stage is expected to reach 5,500 tons. After the completion of the construction, the daily sewage treatment capacity will reach 16,500 tons. The concessionary period is 30 years, and every 5 years is a decision window. After deducting the unit cost (pharmacy fee, utilities, depreciation and amortization, maintenance costs, management expenses, etc.), the sewage treatment unit income per ton of the sewage treatment is RMB 0.49 Yuan. In the first 5 years, the daily guarantee traffic is 8,300 tons. The cost of each stage has been estimated to be 12.44 million Yuan.

(1)The discount rate is based on the benchmark rate of 5 years or more, with a value of 5.9%.

(2)According to the statistical yearbook and the airport master plan, the exponential regression equation for the number of residents and passengers has been constructed. The average annual growth rate of traffic is 16%, while the rate of residents is 6%. The per capita water consumption of the two is about 1:5. As a result, the average annual growth rate of sewage treatment is about 8%.

(3)Refer to the treatment of existing sewage treatment projects in recent years, simulation results show that the average daily traffic is 0.367 tons, and the standard deviation is 0.022. So the volatility is 0.06.

(4)The value of up-movement is 1.66.

(5)The value of down-movement is 1.27.

4.2. Investment decision-making that considers only real options

During the concessionary period, a total of 7 investment strategies are available to investors. The decision-making paths have been shown in figure 1.

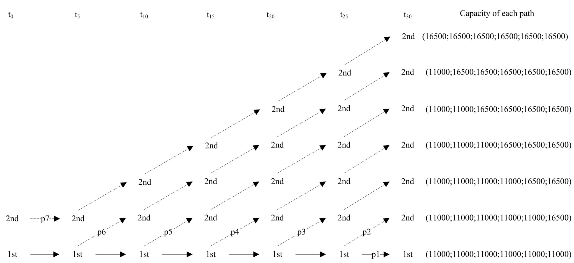

Fig. 1. Decision options in concessionary period

During the 30-year operation period, the sewage treatment traffic and the corresponding probability distribution of the project are shown in figure 2 below.

Consider a non-homogeneous thin symmetric trapezoidal plate of varying thickness and density. The geometry of the plate is shown in Fig. 1.

Take p4 as an example, the upper limit during operation are (11,000; 11,000; 11,000; 16,500; 16,500; 16,500), and actual traffic has been shown in figure 3.

In this case, the income is (unit: RMB in millions):

$$CI = \sum_{t=0}^{30} Q_t \cdot \lambda \cdot 365 \cdot p \cdot (1+r)^{-t} = 26.10 \tag{12}$$

$$Con = 1244 + 1244 * (1+r)^{-t} = 17.70 \tag{13}$$

The NPV in this case is (unit: RMB in millions):

$$NI = CI - Con = 8.40 \tag{14}$$

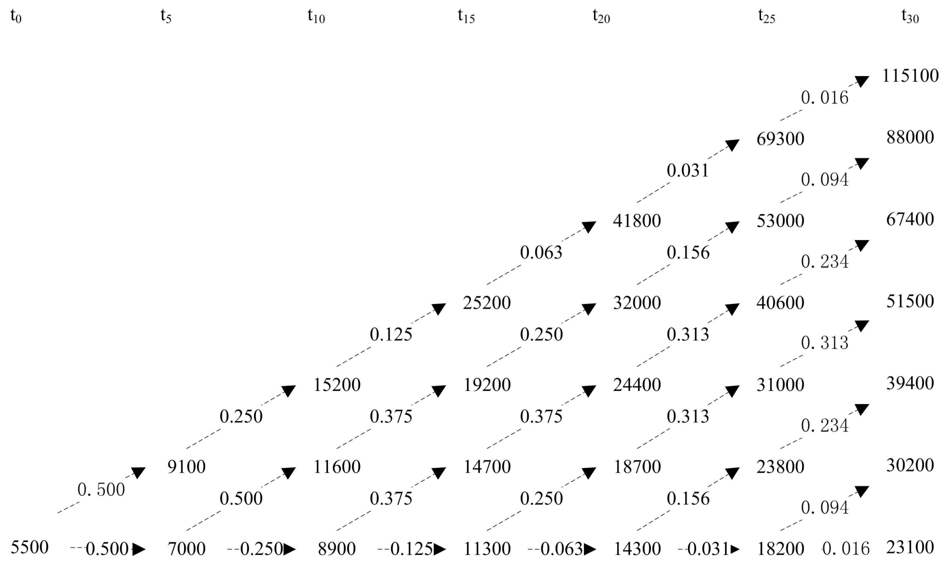

Fig. 2. Expectation of traffic and corresponding probability distribution of the project (unit:t/d)

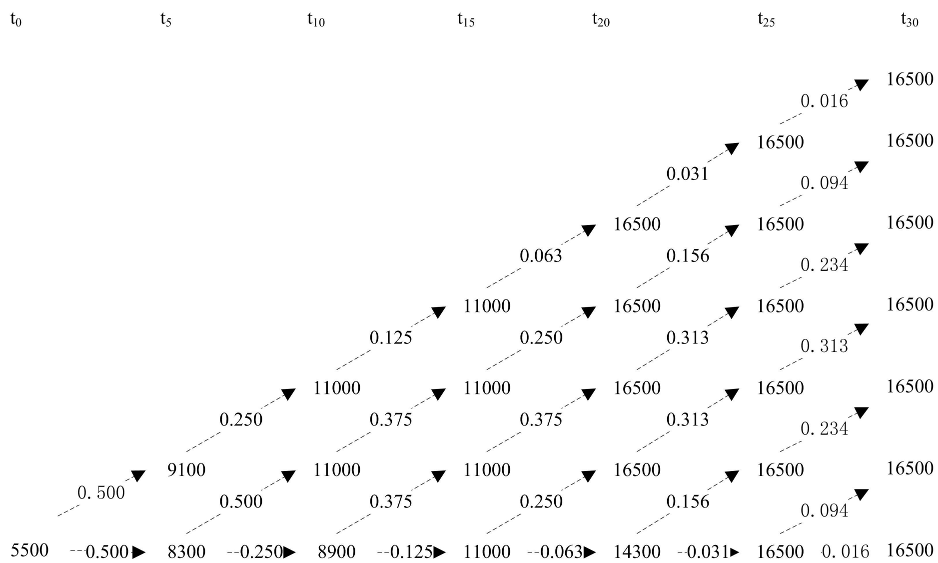

Fig. 3. Actual traffic of decision p4(unit: t/d)

According to the scale of sewage treatment and its corresponding probability distribution at each stage, the net income of each path has been shown in table 1 and figure 4.

Table 1 Project income during the operation period with delayed option (unit: RMB in millions)

Decision	Path	CI	Con	NI
p_1	(1st,1st,1st,1st,1st,1st)	22.52	12.44	10.08
p_2	(1st,1st,1,1st,1st,2nd)	23.41	15.40	8.01
p_3	(1st,1st,1st,1st,2nd,2nd)	24.58	16.39	8.19
p_4	(1st,1st,1st,2nd,2nd,2nd)	26.10	17.70	8.40
p_5	(1st,1st,2nd,2nd,2nd,2nd)	27.68	19.45	8.23
p_6	(1st,2nd,2nd,2nd,2nd,2nd)	28.36	21.78	6.58
p_7	(2nd,2nd,2nd,2nd,2nd,2nd)	28.36	24.88	3.48

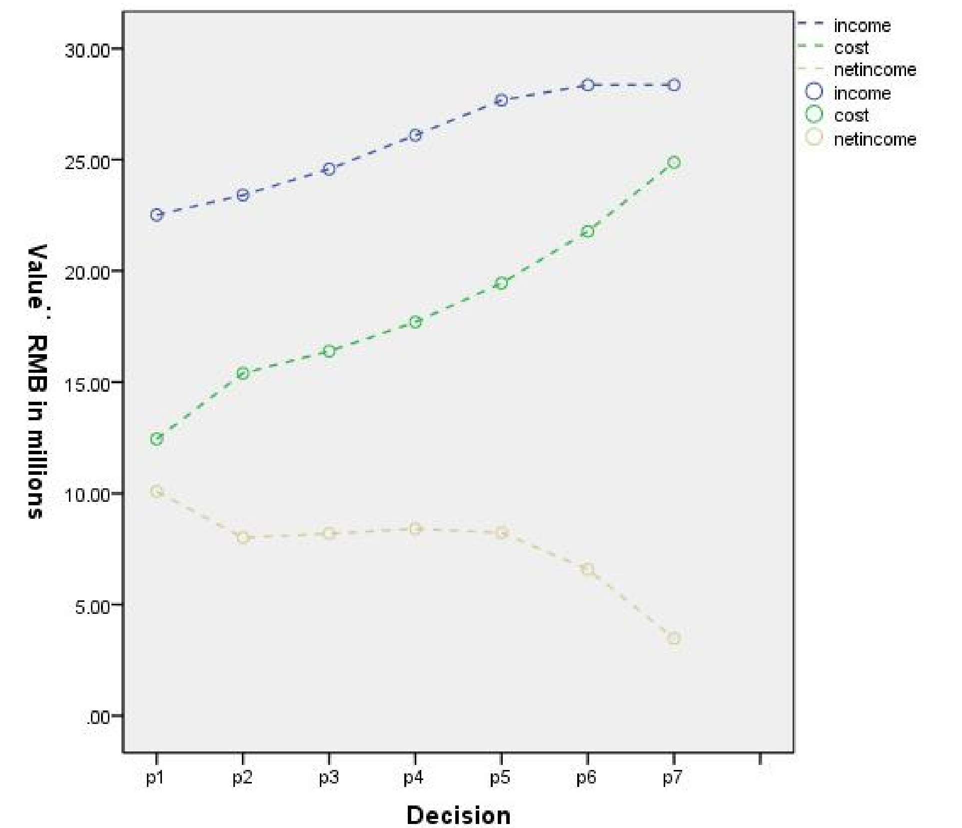

Fig. 4. Project income and cost during the operation period with delayed option

When the private investors have the delay option, taking into account the changes in traffic and the time value of money, the optimal decision is to maintain the scale of the first stage and do not carry out the construction of second stage. Although the income is less than other declines, the cost is reduced more, and the net income has reached its maximum.

4.3. Investment decision-making considering real option and incomplete contract

In the above analysis, it has been found that, without any other restrictions, the instinct of social investors to maximize profits will lead them to choose not to extend or delay expansion. The implementation of emission reduction targets in Hunan province have been affected by certain projects which did not start as scheduled, Therefore, when the actual amount of sewage have exceeded the maximum capacity, it is necessary to make an agreement to prevent the loss of public interest caused by external costs. It should be imposed a certain fee on the part that have exceeded the upper limit of the scale of sewage treatment due to the absence of second construction stage for the ecological compensation.

When the amount of sewage generated is less the design limit, the penalty coefficient is 0. Otherwise, when the traffic has exceeded the planning capacity limit of the first stage and the second stage has still not been carry out, the upper limit of the fee should be determined by the maximum processing capacity after the two stages construction. The net income of the project with different penalty coefficients and decision paths are shown in figure 5.

By introducing the penalty coefficient, there are two effects on the private investors. (**??**)1) When the penalty coefficient changes, the net income of the project will change accordingly. Without exceeding the upper limit of processing capacity, the more waste water is produced, the more fines will be made. (**??**)2) By setting a reasonable penalty coefficient the decision-making path of private investors will change. In the case, when the penalty coefficient is between 0 and 0.3, $p_1$is the optimal decision path, and when the penalty coefficient is between 0.3 and 1, $p_5$is the optimal decision path.

5. Conclusions

Based on the theory of real options and incomplete contracts, this paper discussed the choice of decision-making for private investors in PPP projects in the two kinds of situations. The act of damaging the public interest can be effectively suppressed by introducing penalty coefficients. , According to the analysis of actual cases, the relevant theoretical hypotheses have been verified. The following conclusions and suggestions are obtained.

(**??**)1)The private investors have complete delay options. When the sewage production exceeds the upper limit of processing capacity, the private investors tend to delay the project in order to maximize their own interests. While the ecological damage caused by the outflow of sewage will be borne by the society.

(**??**)2)By introducing the penalty coefficient, it can effectively adjust the decision-making behavior of the private investors even if they have complete delay options. Then the ecological compensation will be achieved. By sensitivity analysis, when the

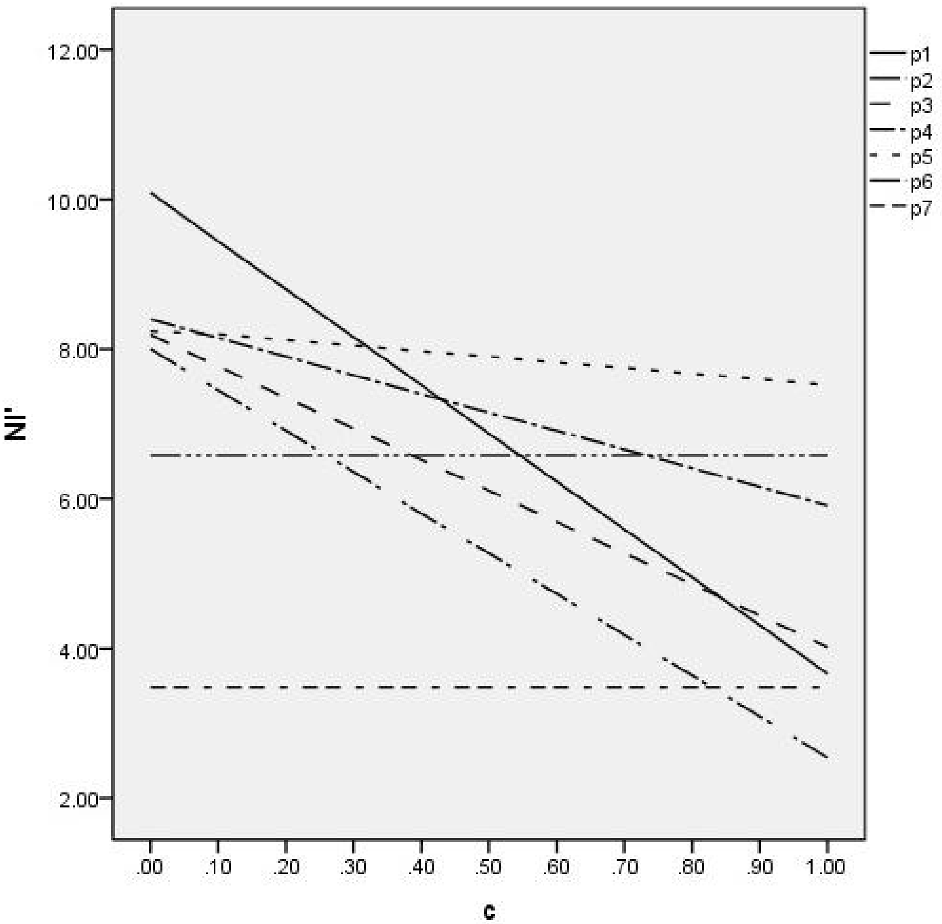

Fig. 5. Sensitivity of the penalty coefficient

penalty exceeds the critical value, it is found that the private investors will choose the expansion to meet requirements. It should be noted that the upper limit of the penalty coefficient is not 1, which means that the penalty can be increased by setting a higher coefficient.

(??)3)In determining the contract, the government should try to determine the corresponding external cost threshold and the elasticity range, and set the trigger mechanism, by which, the decision right should be transferred, if the private investors limit the capacity expansion after the external cost exceeds the threshold.

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